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Techniques of determining latitude in Indian astronomical treatises

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Abstract
The latitude-longitude coordinate system is a well-known method of expressing one's location on the earth's surface. It can be ascertained from references in Indian astronomical treatises that the idea of latitude was well established in this tradition for a long time. It was important for several problems in astronomy, especially for one's location, and was measured using specific techniques and instruments.
This paper brings together some of the chief techniques provided in these texts and classifies them. It is found that the primary method is finding the equinoctial midday shadow using a gnomon and working out the zenith distance from there. Other methods include finding the shadow on any given day with the gnomon and back-computing the latitude, and measuring the polar altitude. In this context, the instruments for this purpose have also been discussed.

Keyword: Latitude, siddhanta, equinoctial midday shadow, Indian astronomy, dhruva

Introduction
The latitude-longitude coordinate system is a common method today of expressing one's location on the earth's surface. References from Indian astronomical treatises indicate that the idea of latitude was well known since the time of the oldest available texts and finds use in much of astronomy. While the core astronomical idea behind this is the same, many different techniques have been discussed in these texts over the course of time for this purpose.
The earliest datable text in the Indian tradition on mathematical astronomy which is available in whole at present is the *Āryabhaṭīya* (499 CE). The era ushered in by this treatise is often called the *Siddhāntic* period (5th century CE onwards), and was a time when several new treatises on astronomy were written. The present study focusses on this period.
While Indian treatises are well acquainted with the concept of latitude, the means of measuring it are mentioned in several places. This includes the procedure itself, as well as the instruments used in the procedure. This paper brings together these methods as a single work, to facilitate further study on these methods.
In the present paper, the astronomical rationale that underpins the process of finding latitude is described first. This is followed by a short account of how latitude is viewed in Indian texts. The methods used in Indian texts are described after this.

Astronomical Rationale
There are several methods to measure the latitude of a place. However, they rely on a common idea: at any given place, the celestial sphere, and by consequence, all celestial objects, are inclined to the north or south by the latitude angle.
At the terrestrial equator, the latitude (ϕ) is 0° and the celestial equator passes through the zenith. As the observer moves northwards, the latitude increases and the celestial equator no longer reaches the zenith; the zenith distance of the highest point of the celestial equator (Where it crosses the meridian) is ϕ. This situation is represented in Figure 1. Similarly, the north pole, which lies on the horizon when the observer is at the equator, is located at an altitude of ϕ for an observer at ϕ.

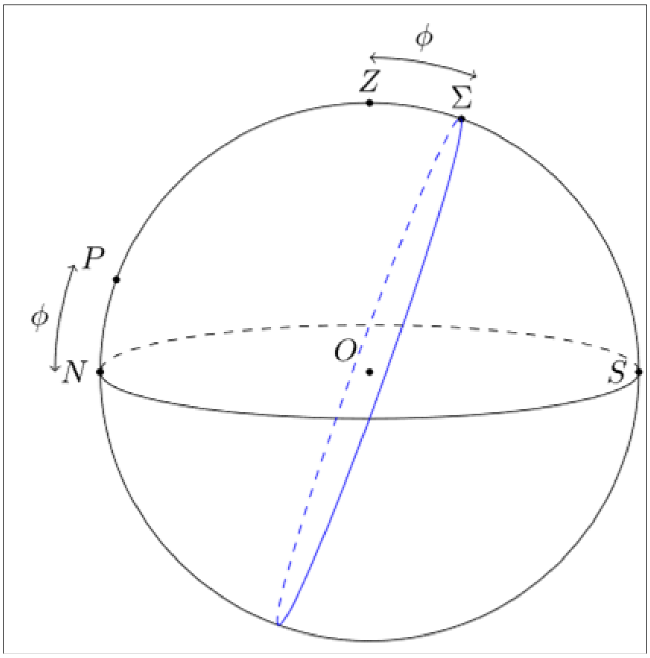

Fig 1: The celestial sphere in the northern hemisphere. NZS is the meridian, the horizontal circle NS is the horizon, the vertical circle with Σ is the celestial equator, Z is the zenith and P is the pole. The latitude is ϕ.

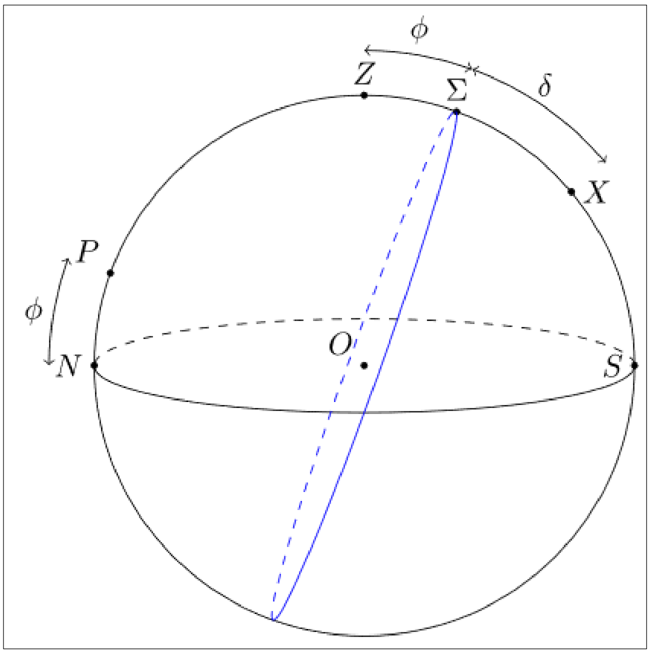

Fig 2: The celestial sphere showing an object located at X. The object is on the meridian line NZS and has declination δ.

Consider the situation represented in Figure 2. Let an object be located on the meridian line and have declination δ (represented as X). In such a case, from the figure:

$$z = \phi - \delta$$

Or:

$$\phi = z - \delta$$

Hence, if the declination of an object is known, the latitude can be computed by measuring the zenith distance when the object transits the meridian. Based on this concept, it is possible in theory to use any object to measure the latitude. However, in practice, only some objects, such as the sun, are used for this purpose.

The sun is one of these objects. The shadow of the sun as cast by a gnomon is measured and the zenith distance is found from this. This measurement may be taken at different points in time.

1. **Equinox:** During equinoxes, the sun is at or very close to the equator. This implies that: $\delta \approx 0$. Therefore, zenith distance is nearly equal to latitude.
2. **Solstice:** Here, the sun has maximal declination, and this value is known (ε).
3. **Any given day:** On all other days, the declination of the sun can be predicted from the longitude λ using the equation:

$$\sin\delta = \sin\lambda \sin\varepsilon$$

In cases 2 and 3 above, the declination is used to derive the latitude. If the declination is towards the south, it is negative, and if to the north, it is positive. The equation can be used easily to find the latitude.

The other common object used in this context is the pole star. Due to the precession of equinoxes, the star at the pole changes over the course of centuries. When a particular star comes near the north celestial pole, it would be called the pole star, such as the present-day Polaris. Such stars are particularly useful when finding latitude as the altitude of the pole is equal to the latitude. Therefore, measuring the pole star's altitude is a good estimate for the latitude, minimizing the required calculations.

Deriving latitude from the moon and planets is theoretically possible, as it relies on the same principles as those enumerated above. However, these are not explored here as these methods are not as common as the previous ones.

Latitude in Indian astronomical literature

The knowledge of the Earth being a sphere is an important prerequisite for understanding latitude, and this was known to Indians. The *Siddhāntic* era authors were also aware of latitude, as mentioned above. The earth's axis is called *akṣa* (Which means axis or axle). As one moves to the north from the equator, the axis is elevated with respect to its position when one is at the equator. This altitude which is nothing but the latitude, is called *akṣāṁśa* or simply *akṣa*. Obtaining knowledge of *akṣa* is one of the three questions (*Tripraśna*), which form part of the core discussion in the majority of *Siddhāntic* works.

The elements required to determine latitude such as the celestial sphere, equator and the poles are all described from the time of *Āryabhaṭīya*. The methods of latitude measurement used by the *Siddhāntic* astronomers, are explored below.

Methods

Equinoctial midday shadow

In the Indian astronomical tradition, the preeminent method to determine latitude is using the gnomon or *śaṅku*. The *śaṅku* is a very simple instrument as it comprises of a vertical pole fixed into the ground. The standard height of the gnomon was 12 *aṅgulas* (About 22 cm). In general, it is the sun's shadow (*Chāyā*) that was measured and from this, the latitude is derived.

Sūryasiddhānta mentions that the astronomer must evaluate the locations of various celestial circles and other factors before making observations. One of the factors mentioned here is the equinoctial midday shadow. The treatise states (12b-13a):

एवं विषुवती छाया स्वदेशे या दिनार्धजा ।।
दक्षिणोत्तररेखायां सा तत्र विषुवत्प्रभा ।

Evaṃ viṣuvatī chāyā svadeśe yā dinārdhajā ||
dakṣiṇottararekhāyāṃ sā tatra viṣuvatprabhā |

The shadow cast at midday at one's own place, on the equinoctial day, on the north-south line, is the equinoctial shadow (*Viṣuvatprabhā*) of that place.

The shadow of the gnomon gives the zenith distance of the sun at any given time. However, if it is measured on the day of the equinox, the sun will be on or near the celestial equator. If this measurement is taken when the sun is at the meridian, i.e., during midday, the zenith distance will be nearly equal to the latitude. The shadow of this day has a special name: *palabhā*. Here *pala* means latitude and *bhā* refers to the shadow. Some synonyms of this word include *viṣuvacchāyā* and *akṣacchāyā*. This is one of the most common methods used to find latitude and finds mention in nearly all texts.

One variant to this method is also found in the *siddhāntas*. Certain authors such as Lalla (c. 750 CE) ask the reader to find the shadow during the vernal and autumnal equinoxes and find their arithmetic mean. This is considered as the final shadow which is used for calculating latitude. Lalla states (*Śiṣyadhīvṛddhidatantra*, 1981, 4.4a) [13]:

भानौ गते क्रियतुलादिमहर्दले ये च्छाये तयोर्युतिदलं विषुवत्प्रभा स्यात् ।

Bhānau gate kriyatulādimahardale ye cchāye tayoryutidalaṃ viṣuvatprabhā syāt |

One half of the sum of the length of the two midday shadows of the gnomon, when the sun is at the first point of Aries and Libra, is the *palabhā*, or 'equinoctial midday shadow.' (Chatterjee, 1981: 64) [2]

This method is not as common as the one above but was still mentioned by Lalla (Quoted above), Āryabhaṭa II (c. 950 CE), Śrīpati (c. 1050 CE, Pejathaya, 2016: 87) [8], and the author of *Grahaṇamukura* (Shylaja *et al*., 2024) [12]. This is further discussed later in the paper.

Shadow (On other days)

In some cases, it may also be particularly convenient to find the sun's shadow length on any given date and compute the latitude from that. It is necessary to know the sun's position (*Sphuṭa*) and the precession of the equinoxes (*ayanāṁśa*) to do this computation. If these are known, it enables the astronomer to conduct the observation immediately rather than having to wait several months for an equinox.

The *Sūryasiddhānta* states (3.14b-17a):

मध्यच्छाया भुजस्तेन गुणिता त्रिभमौर्विका ।।
स्वकर्णाप्ता धनुर्लिप्ता नतास्ता दक्षिणे भुजे ।
उत्तराश्चोत्तरे याम्यास्ताः सूर्यक्रान्तिलिप्तिकाः ।।
दिग्भेदे मिश्रिताः साम्ये विश्लिष्टाश्चाक्षलिप्तिकाः ।
ताभ्योऽक्षज्या च तद्वर्गं प्रोज्झ्य त्रिज्याकृतेः पदम् ।।
लम्बज्यार्कगुणाक्षज्या...

Madhyacchāyā bhujastena guṇitā tribhamaurvikā ||
Svakarṇāptā dhanurliptā natāstā dakṣiṇe bhuje |
Uttarāścottare yāmyāstāḥ sūryakrāntiliptikāḥ ||
Digbhede miśritāḥ sāmye viśliṣṭāścākṣaliptikāḥ |
Tābhyo'kṣajyā ca tadvargaṃ projjhya trijyākṛteḥ padam ||
Lambajyārkaguṇākṣajyā.

Keeping the midday shadow as the base, multiply it by *trijyā*, and divide it by the corresponding hypotenuse. This is converted to the arc and is the sun's zenith distance in minutes. When the base is towards the south, the zenith distance is to the north, and when to the north, it is to the south. Take the sun's zenith distance and its declination. If these are in opposite direction, find their sum; if in the same direction, find their difference. This gives the latitude. The Rsine of the latitude is squared and subtracted from the square of *trijyā*. The square root of this is the Rsine of the colatitude.

Verses 3.14b and 3.15 are concerned with deriving the zenith distance of the sun from the midday shadow. This procedure relies on the similar triangles and is therefore based on proportions.

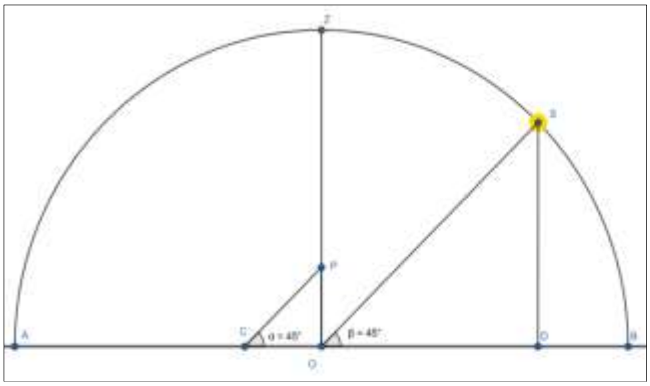

Fig 3: A representation of the shadow cast by a gnomon (S being the sun and OP being the gnomon).

Figure 3 depicts the shadow cast by a gnomon. As the sun is very distant, angle SOD and angle PCO are nearly equal, as depicted in the figure. This implies that POC and SOD are similar triangles.

The verse states that the shadow length (OC) must be kept as the base. This must be multiplied by *trijyā* (the standard radius, equal to OS in the diagram) and divided by the corresponding hypotenuse (PC). The result will be OD. This must be converted to arc, i.e., the arcsine of this line must be found. This will be equal to arc ZS, which is the zenith distance. This will be towards the north when the shadow points south and vice versa. The rest of the verses specify the process of computing latitude from zenith distance and declination.

Polar altitude

The last method is mentioned in several texts on astronomy during the *Siddhantic* period. This involves finding the altitude of *dhruvatārā* or the pole star in the sky. This directly yields the latitude and there is no need for any further computation.

Bhāskara II mentions the pole star in *Siddhāntaśiromaṇi* (1981, *Tripraśnavāsanā 33*) [13] as follows:

यन्त्रवेधविधिना ध्रुवोन्नतिर्या नतिश्च भवतोऽक्षलम्बकौ ।
तौ क्रमाद्विषुवदह्नयहर्दले येऽथवा नतसमुन्नता लवाः ।।

Yantravedhavidhinā dhruvonnatiryā natiśca bhavato'kṣalambakau |
Tau kramādviṣuvadahnyahardale ye'thavā natasamunnatā lavāḥ ||

The altitude and zenith distance of the pole [star] as found from observation using a *yantra* (Instrument) gives the latitude and colatitude. Alternatively, at midday on the day of the equinox, zenith distance and altitude of the sun give the same [result].

Hence these were understood to be equivalent. The procedure to observe the polar altitude is given by Varāhamihira in *Pañcasiddhāntikā* (1993, 13.30-33) as follows:

सलिलेन समं कृत्वा तुङ्गं फलकं यथादिशं दृष्ट्वा ।
दक्षिणकोट्यां शङ्कुं फलकप्रमितं व्यवस्थाप्य ।।
ऋजुशङ्कुबुध्नविन्यस्तलोचनो नामयेत्तथा शङ्कुम् ।
भवति यथा शङ्क्वग्रं ध्रुवतारादृष्टिमध्यस्थम् ।।
पतितेन भवति वेधो लङ्कायामूर्ध्वगेन तु सुमेरौ ।
विनतेन चान्तराले फलके चाक्षोर्ध्वसूत्रसमम् ।।
तत्रावलम्बको यः सोऽक्षज्या तस्य शङ्कुविवरं यत् ।
विषुवदवलम्बकोऽसौ याम्योत्तरदिक्प्रसिद्धिकरः ।।

Salilena samaṃ kṛtvā tuṅgaṃ phalakaṃ yathādiśaṃ dṛṣṭvā |
Dakṣiṇakoṭyāṃ śaṅkuṃ phalakapramitaṃ vyavasthāpya ||
Ṛjuśaṅkubudhnavinyastalocano nāmayettathā śaṅkum |
Bhavati yathā śaṅkvagraṃ dhruvatārādṛṣṭimadhyastham ||
Patitena bhavati vedho laṅkāyāmūrdhvagena tu sumerau |
Vinatena cāntarāle phalake cākṣordhvasūtrasamam ||
Tatrāvalambako yaḥ so'kṣajyā tasya śaṅkuvivaraṃ yat |
Viṣuvadavalambako'sau yāmyottaradikprasiddhikaraḥ ||

An elevated board must be placed in alignment with the directions, on a place that has been confirmed to be flat with water. On the southern end, a *śaṅku* must be affixed such that its size (Length) is the same as that of the board (In a north-south direction). It must then be lowered so that the pole star is observed at the end of the *śaṅku*. If it falls (i.e., is lowered completely), (The observer) is at *Laṅkā* (The equator) and if it points upwards, (He is) at *Sumeru* (The north pole). The perpendicular dropped (From the tip of the *śaṅku* onto the board) is the R-sine of the latitude. The difference between the perpendicular and the gnomon amounts to the R-cosine of the latitude. The R-cosine line follows the north-south direction.

The above verses indicate that a *śaṅku* must be placed on an elevated and flat base, and the observer must bend the *śaṅku* such that it lies on the line of sight to the pole star. Śrīpati also mentions the *dhruva* and the method to measure its altitude in *Siddhāntaśekhara* (1947, 4.122-123) [14]. The description he offers closely follows that of Varāhamihira. However, in Śrīpati's description, the observer must view the celestial north pole (*dhruva*) from the edge of this base, passing his line of sight through the tip of the *śaṅku*.

Sūryasiddhānta (1935, 12.72) and Bhāskara I (in *Āryabhaṭīyabhāṣya* under *Gola 16*) mention the *dhruva* and its altitude. Lalla (*Śiṣyadhīvṛddhidatantra* 4.2), Vaṭeśvara (*Vaṭeśvara-siddhānta* 3.4, 3.26) and several other astronomers.

The method of polar altitude is specifically applicable only at a time when a star is located near the north celestial pole, such as in the present day with Polaris. Due to the precession of equinoxes, Polaris would not have been at that location during the time of the above astronomers. During Śrīpati's time, Polaris was still around 6° from it, hence it was still far away. If that were used in this observation, the result would have been highly inaccurate. During Varāhamihira's time, this gap was even larger, and the wobble of Polaris around the pole (i.e., a visible difference between the upper and lower culminations) would have also been very apparent. Hence, this is unlikely to have been the star described here.

It may be noted in this context that several Indian texts such as some Vedic texts and the *Purāṇas* refer to a fixed star in the region of the pole, naming it '*dhruva'*. Iyengar (2011) [3] connects this star to α-Draconis (Thuban), which was closest to the celestial pole in the early third millennium BCE.

The relationship of the *Siddhāntic dhruva* to that of the *Purāṇas*, and to stars in the region of the pole needs further investigation. The present authors have compiled several references pertaining to the pole star that occur in Indian astronomical literature in a separate paper (Iyer & Pejathaya, 2024) [4], and provided a preliminary analysis of the content.

Instruments

Apart from the gnomon, there were other instruments used in the Indian tradition for measuring latitude. A list of some of these follows, along with a brief account of each:

1. ***Cakra yantra* (Circle):** Mentioned by Brahmagupta and other astronomers, this instrument comprises of a graduated circular plate with an axis. The plate is placed vertically such that its axis casts a shadow. The angle between the downward direction (Indicated through a plumbline) and the shadow indicates the sun's zenith distance. (Sarma, 2019: 3299) [9]
2. ***Dhanuryantra* (bow):** This is like the *cakrayantra* but comprises of only half a circle. (Sarma, 2019: 3299) [9]
3. ***Turīya yantra* (Quadrant):** This is also like the *cakrayantra* but comprises of only a quarter of a circle. (Sarma, 2019: 3299) [9]
4. ***Dhīyantra*:** Bhāskara II describes this instrument in his *Siddhāntaśiromaṇi*. He explains the procedure to obtain the latitude by the observation of the pole star with this instrument (Ohashi, 1994: 235) [7].
5. **The triangle instrument:** This is an instrument mentioned by *Vaṭeśvara* and simply called *yantra* (meaning instrument). It is shaped as a right-angled triangle. When the hypotenuse is directed to the pole star, the base is equal to the length of the gnomon and the upright is equal to the equinoctial midday shadow (Ohashi, 1994: 226) [7].
6. ***Yantrarāja* (Astrolabe):** The earliest Sanskrit references to the astrolabe come from during the Delhi Sultanate in North India (c. thirteenth century CE). The astrolabe was used for several purposes including measurement of latitude. The versatility of this instrument has been praised in several sources, because of which it acquired the name *yantrarāja* or 'king of instruments.' (Sarma, 2019) [9]

Discussion

From the above information, it is apparent that there were primarily four methods to determine the latitude of a location:

1. Finding the equinoctial midday shadow(s)
2. Finding the shadow on other days
3. Measuring the polar altitude

A disadvantage of the first method is that the instant of equinox (When the sun is exactly on the equator) will seldom match the moment of midday when the shadow is taken. The variant proposed by Lalla and other astronomers is clearly an attempt at remedying this. However, both these methods are difficult to implement as the astronomer needs to wait several months to get a result. The major advantage of the second method is that the procedure is applicable on all days. Therefore, an astronomer need not wait for several months (For one equinox) or even a year (For two equinoxes) to take measurements of the shadow. A disadvantage is that it requires the sun's declination to be known accurately, which relies on having good models of planetary motion at one's disposal. The precession of equinoxes would also have to be understood well. The measurement of polar altitude presents an independent alternate which would give direct results. However, this can only be used when there is a star at or very close to the pole.

The idea of repeating measurements of two equinoxes, finding the average of the shadows and then computing latitude is clearly intended to improve accuracy. This method will improve accuracy specifically when the equinoxes occur on either side of midday. In other cases, there is a chance that the average gives a more inaccurate result than what would have been got when only one equinox was used. The ideal case would be to choose a day when equinox occurs very close to midday.

Most treatises prescribe multiple methods, if not all the ones above. It is possible that the authors intended that an astronomer should use all of them to verify the value of latitude arrived. This tendency is observed in case of several computations and observations where *siddhāntas* often describe multiple methods to arrive at the result.

Conclusion

Latitude is a concept that is well entrenched in Indian *Siddhantic* treatises. Multiple methods can be identified for finding latitude. All of these are based upon the same core idea of finding the altitude of celestial objects. However, different methods were worked out for this:

Of these, the most prominent is finding the equinoctial midday shadow (Either for one equinox or the average of two equinoxes). An alternate is finding the shadow on some other day. Both these yield the zenith distance of the sun from which it is possible to calculate the latitude. Another common method is finding the altitude of the pole star. This star is unlikely to be Polaris owing to the latter's distance from the pole, but its identity is not certain. To work out the angles, different instruments such as the *śaṅku* (Gnomon), *cakrayantra* and variants, *dhīyantra*, and, in later times, the astrolabe (*Yantrarāja*), were also used.

The above account brings together pieces of information pertaining to the empirical determination of latitude from different sources and puts them into separate categories. Being part of *tripraśna* or the three questions which most *siddhāntas* sought to answer, this is an indicator of the observational techniques that Indian astronomers tried to develop. It is hoped that this work will facilitate further exploration of this area.

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