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Implication of geometrical aspects in *Bhujakoṭikarṇanyāya*

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Kerala mathematics became a well developed branch of knowledge system during 14th century. The novel approach put forth by the Kerala mathematicians have got special attention among the scholars all over the world. Among those, geometrical treatment of algebraical identities has much significance. The fact is that the Kerala mathematicians arrived at the results with the help of geometry. Thus a beautiful blend of geometry and algebra can be seen in the mathematical texts like *Āryabhaṭīyabhāṣya* of Nīlakaṇṭha Somayājin, *Siddhāntadarpaṇa* of Nīlakaṇṭha Somayājin, *Yuktibhāṣā* of Jyeṣṭhadeva and *Kriyākramakarī* commentary of *Līlāvatī* by Śaṅkara Vāriyar and Nārāyaṇan Nampūtiri. This paper tries to mark this geometrical feature of Kerala mathematics by analyzing *Bhujakoṭikarṇanyāya* in the above mentioned texts.

Bhujakoṭikarṇanyāya

Bhujakoṭikarṇanyāya deals with the measurement of the sides and the hypotenuse of a right-angled triangle. If a and b are the two sides of a right –angled triangle, then the third side 'c' i.e, hypotenuse can be found out as c=square root of $(a^2 + b^2)$ using modern notations. From the period of *Śulbasūtra* itself, ancient Indians had often made the discussions on the relation between the sides and the diagonal of right angled figures. Later on, one of the medieval Indian mathematical texts like *Līlāvatī* of Bhāskara II simply presents this idea in the *Kṣetravyavahāra*.

इष्टाद् बाहोर्यः स्याद् तत्समतिर्यग् दिशीतरो बाहुः।
त्र्यश्रे चतुरश्रे वा सा कोटिः कीर्तिता तज्ज्ञैः।।135।।
तत्कृत्योर्योगपदं कर्णो, दोःकर्णवर्गयोर्विवरात्।
मूलं कोटिः, कोटिश्रुतिकृत्योरन्तरात् पदं बाहुः।।136।। [1]

[In right-angled triangle, one of the sides is called base and the side perpendicular to it is called the altitude. The same terminology holds for a rectangle.

In a right-angled triangle, square root of the sum of the squares of the base and the altitude is the base and the altitude is the hypotenuse and the square root of the difference between square of the hypotenuse and the base (resp. altitude) is the altitude (resp. base) [2].

But the Indian mathematicians upto Bhāskara II did not discuss the proofs of these sūtras. Later on, by studying the Keralite texts, one can seen the detailed proofs of this.

Bhujakoṭikarṇanyāya in *Siddhāntadarpaṇa*

Nīlakaṇṭha Somayājin from Śrīkuṇḍapura (now it is known as Tṛkkaṇṭiyūr near Tirur in Malappuram District) is the author of the astronomical text *Siddhāntadarpaṇa*.

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1 *Līlāvatī of Bhāskarācārya with Kriyākramakarī of Śaṅkara and Nārāyaṇa*, (Ed.) K.V. Sarma, p.277.

2 *Līlāvatī of Bhāskarācārya- A treatise of Mathematics of vedic tradition*, (Trans.) Krishnaji Patwardhan and et.al, pp.113-114.

He has also written a commentary on *Siddhāntadarpaṇa*. It is written in prosaic style. The editor of *Siddhāntadarpaṇa*, K.V.Sarma points out that Nīlakaṇṭha's discursions are often highly instructive. [3] He also cites the explanation of *Bhujakoṭikarṇanyāya* as an instance to show its instructive nature.

Nīlakaṇṭha Somayājin clearly interprets how the sides of the resulting geometrical figure obtained from taking two squares of having lengths equal to *bhuja* and *koṭi* respectively, will be equal to the desired *karṇa* i.e hypotenuse.

1. Place the *bhuja* square (small square) adjacent to the *koṭi* square in such a way that their corners coincide as in fig 1.

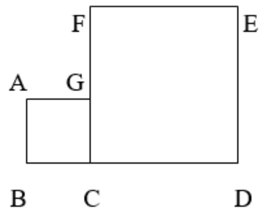

Fig 1

[ABCG= *bhuja* square and CDEF= *koṭi* square]

Then BD= *bhuja+ koṭi*, EF=CD=*koṭi*, AG=AB= *bhuja* and FG=*koṭi- bhuja*

Thus there are six *paryantabhāgas* for the resuting figure. (एवं च पर्यन्तभागाः षोढा विभक्ताः स्युः ।)[4]

2. Mark a point on the large square at distance equal to the difference of *koṭi* and *bhuja*. The other sides are equal to *bhuja* and *koṭi*. This larger square can be cut along this mark, we get one rectangle of length '*koṭi*' and breadth '*bhuja*'.(Fig 2)

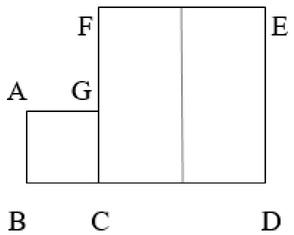

Fig 2

The remaining two sides are of lengths equal to *koṭi*, two are of lengths equal to *bhuja* and the other two of lengths of the difference between *koṭi* and *bhuja*.

महच्चतुरश्रं लाञ्छनमार्गेण छिन्द्यात्। तदा तदायतचतुरश्रदोःसमविस्तारं कोटिसमायामं च, इतरस्य खण्डस्याश्रे द्वे कोटिकोटितुल्ये, द्वे चतुरश्रतुल्ये, अन्ये दोःकोट्यन्तरतुल्ये ।[5]

3. Cut it from the top, then a square of lengths equal to the difference between *koṭi* and *bhuja* and another rectangle of length equals to *koṭi* and breadth equals to *bhuja* are obtained.
4. Cut those rectangles diagonally will give four triangles. Place them in such a way that the *karṇas* of the figure outside. Lastly, fill the gap by the square having the length of the difference between *koṭi* and *bhuja*.

Similar type of explanation is given in the *Āryabhaṭīyabhāṣya* of Nīlakaṇṭha Somayājin. It is quoted by him in the work *Siddhāntadarpaṇa*. He also explains the case of two squares in equal dimension.[6]

Bhujakoṭikarṇanyāya in Yuktibhāṣā

Yuktibhāṣā uktibhāṣā of Jyeṣṭhadeva is a keralite mathematical text written in 17th century A.D. It is considered as the first text written in old Malayalam language. In *Yuktibhāṣā, Bhujakoṭikarṇanyāya* is explained during the discussion of *paridhi-vyāsaprakaraṇa*. i.e the relationship between circumference and diameter of a circle. For this, first the author of *Yuktibhāṣā* illustrates the basic rule "the square of the *bhuja* and *koṭi* will give the square of the karṇa." Algebraically, it can be written as $bhuja^2+ koṭi^2= karṇa^2$.Here *Yuktibhāṣā* describes this in geometrical way.[7]

1. Consider two squares having the length of koṭi and bhuja respectively. (For convenience, take the lengths as 'a' and 'b')
2. Join these two squares as in fig 3.

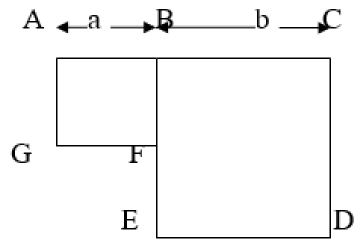

Fig 3

ABGF is the bhujakṣetra of side 'a' and BCDE is the koṭikṣetra. Then the western side of the square ABCG will remain as vacant.

3. Mark a point 'O' which is equal to the length 'a' from B. Then the length OD is found to be 'a'.(fig4)

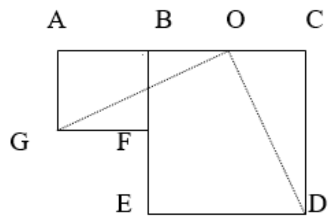

Fig 4

4. Then from the point O, draw OG and OD.
5. Cut the triangles OAG and OCD and join as shown in fig 5.

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3 *Siddhāntadarpaṇa of Nīlakaṇṭha Somayāji with auto-commentary*, (Ed.)K.V.Sarma,p.xii.

4 ibid.,p.23.

5 idem.

6 ibid., p.24.

7 *Yuktibhāṣā* (Part I),(Ed.)Ramavarma Maru Tampuran and A.R.Akhileswara Iyer,pp.62-64.

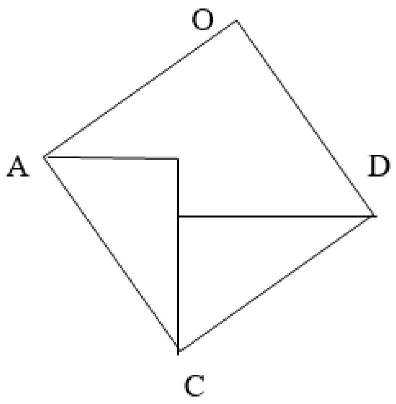

Fig 5

The new figure will be a square OACD. Thus it is proved that the side of the new square is equal to the sum to the squares of *bhuja* and *koṭi*.

Bhujakoṭikarṇanyāya in *Kriyākramakarī*

Kriyākramakarī is a Keralite Sanskrit commentary on the *Līlāvatī* of BhāskaraII. Its first 199 verses is commented by Śaṅkara Vāriyar of Tṛkkuṭaveli (now it is known by the name Tṛkkaṭīri near Ottappalam) and completed by Nārāyaṇan Nampūtiri of Mahiṣamaṅgalam. It is one of the elaborate exposition on the text *Līlāvatī*. Śaṅkara Vāriyar strictly follows the geometrical tradition of Kerala School of mathematics.

In *Kriyākramakarī*, Śaṅkara Vāriyar explains the geometrical aspect of *Bhujakoṭikarṇanyāya* in verses. He has written almost sixty verses to describe the geometrical treatment. He demonstrates the *Bhujakoṭikarṇanyāya* in two ways.

First method

1. Consider two squares of sides 'a' as in fig6.

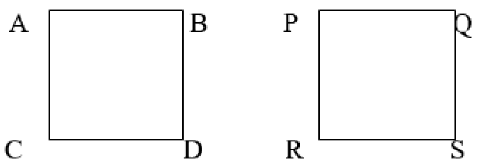

Fig 6

2. Cut PQRS diagonally, we get four equilateral triangles. (fig7)

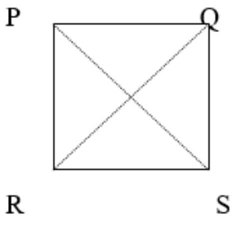

Fig 7

3. Place each of them on the sides of the square ABCD. (fig8)

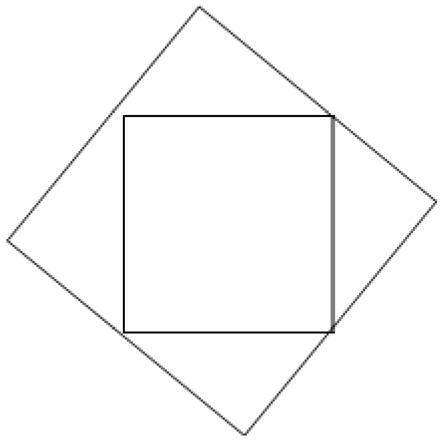

Fig 8

The length of the new square EFGH is equal to the length of the diagonal PR and QS. [8] This is very similar to the explanation in *Siddhāntadarpaṇa*.

Second method

1. Let ABCD and MNOP be the two squares of lengths equals to Koṭi 'a' and bhuja 'b' (fig9).
2. Join these two squares and mark a point 'Q' equal to the length of the bhuja 'b' from the koṭi square.

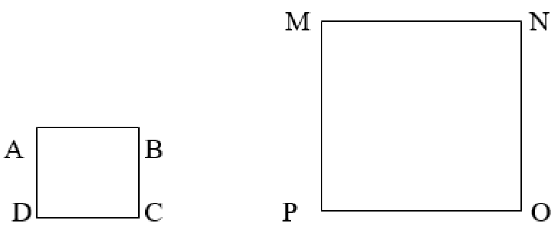

Fig 9

3. Cut along the dotted portion, we obtain two triangles and arrage them as shown in fig 11.

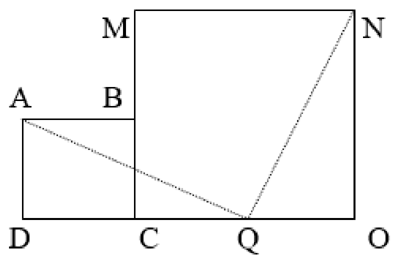

Fig 10

This will be a square of length equals to hypotenuse. This method is also explained in *Yuktibhāṣā*.

Conclusion

From the above cited examples, one can infer that Kerala School of mathematics definitely follows the way of

[8] *Līlāvatī of Bhāskarācārya with Kriyākramakarī of Śaṅkara and Nārāyaṇa*,(Ed.) K.V.Sarma,p.281.

expressing the mathematical results using geometrical approaches. Nīlakaṇṭha Somayāji, one of the leading mathematicians of the Kerala school, frame a firm basis to this view through his works *Āryabhaṭīyabhāṣya* and *Siddhāntadarpaṇa*. He says that एतत्सर्वं युक्तिमूलमेव, न त्वागममूलम्[9]. And also the disciples of Nīlakaṇṭha like Jyeṣṭhādeva and Śaṅkara Vāriyar tries to keep the tradition in a wider sense.

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[9] *Siddhāntadarpaṇa of Nīlakaṇṭha Somayāji*, (Ed.) K.V.Sarma,p.24.