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Some sequences with Vedic matrix relations

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Abstract
The paper deals with some Sutras and Sub-sutras related to classical sequence in Vedic mathematics with meaning, examples and matrix relations.

Key words: Sequence spaces, matrix transformation, vedic mathematics, sutras

1. Introduction

A sequence is a set of numbers defined as a function and a sequence space is a linear space of functions defined on the set of counting numbers. The idea of the sequence spaces was motivated through the classical results of summability theory which were first introduced by Italian Mathematician Emesto Cesaro (1859 – 1906), French Mathematician Emile Borel (1871 – 1956), Danish Mathematician Niels Erik Norlund (1885 – 1981) and others. The summability theory deals with the most general linear operator which transforms a sequence space into another sequence space in terms of an infinite matrix. The matrix transformations are of methods in abstract sense was done by the celebrated German mathematician Otto Toeplitz (1881 – 1940) in 1911 who used to characterize matrix transformation and gave the necessary & sufficient conditions for an infinite matrix to be regular. The regular in the sense that it preserves the limit of the convergent sequences.

Vedic Mathematics is the ancient system of mathematics which provides multi-dimensional thinking in human brain, based on sixteen sutras and thirteen sub-sutras. The sixteen sutras in the present work is based from part of a Parisista of the Atharvaveda. One such popular topic of Hindutva imposition was Vedic Mathematics. Much of the type about this topic is based on a book [5] authored by the Sankaracharya Jagadguru Swami Sri Bharati Krisna Tirthaji Maharaja titled Vedic mathematics published in the year 1965 and reprinted several times since the 1990s. The entire field of Vedic Mathematics is supposedly based on sixteen sutras in Sanskrit, which they claim can solve all modern mathematical problems.

So, there is a good scope to study and inter relate the classical notion of sequence space with the ancient mathematics relative to Vedic. The objective of the paper is to discuss some Sutras and its Sub-sutras with meaning, examples and possible matrix relations.

2. Main Results

We have the following sixteen sutras, its meaning and sub-sutra with examples [5] pp XVII-XVIII. Here, the symbols have their usual meanings.

2.1. The first sutra is "एकाधिकेन पूर्वेण",

whose english translation is "Ekādhikena Pūrvena".
Its meaning is "By one more than the previous one."
As an example, we get 1, 1+1, 2+1, 3+1, 4+1, 5+1, 6+1, 7+1, 8+1 i.e. 1, 2, 3, 4, 5, 6, 7, 8, 9.
The first row consists of 1, 2, 3, 4, 5, 6, 7, 8, 9 and the second row is 2, 4, 6, 8, 1, 3, 5, 7, 9.
If we add the first and the last numbers in each row or column, we get
10, 11, 12, 13, 14, 15, 16, 17, 18 which represents a sequence.
The Sub-sutra is "आनुरूप्येण", whose english translation is "Ānurūpyena".
Its example in Vedic Matrix is a nine by nine square array of numbers formed by taking a multiplication table and replacing each number by digit sum as follows:
10 becomes 1, 11 becomes 2, 12 becomes 3, 13 becomes 4, 14 becomes 5, 15 becomes 6, 16

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becomes 7, 17 becomes 8, 18 becomes 9, that is 1, 2, 3, 4, 5, 6, 7, 8, 9
and 1+8 = 9; 2+7 = 9; 3+6 = 9; 4+5 = 9, then ordered pairs (1, 8), (2, 7), (3, 6), (4, 5) are sequential matrices.
Its Vedic matrix relations are as follows:
(1, 8): $(1 \times 6) + (8 \times 6) = 6 + 48 = 54$ and $1 + 8 = 9$.
(2, 7): $(2 \times 6) + (7 \times 6) = 12 + 42 = 54$ and $2 + 7 = 9$.
(3, 6): $(3 \times 12) + (6 \times 12) = 36 + 72 = 108 = 2(54)$ and $3 + 6 = 9$.
(4, 5) : $(4 \times 6) + (5 \times 6) = 24 + 30 = 54$ and $4 + 5 = 9$.
For 9s : $9 \times 21 = 189 = 1 + 8 + 9 = 18 : 1 + 8 = 9$.

2.2. The second sutra is "निखिलं नवतश्चरमं दशतः"
whose english translation is "Nikhilam Navataścaramam Daśatah".
Its meaning is "All from 9 and the last from 10".
For example, we consider the product 98×95. Then, we get

```
  98 - 02
× 95 - 05
─────────
  93 / 10
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Its Sub-sutra is "शिष्यते शेषसंज्ञः" whose English translation is Śisyate Śesamjnah.

2.3. The third Sutra is "ऊर्ध्वतिर्यग्भ्याम्"
whose english translation is "Ūrdhva Tiryagbhyām".
Its meaning is "Vertically and cross-wise."
For example, we consider the product 56×74. Then,

```
    56
×   74
─────────
41_6 4_2 4
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Its sub-sutra is "आधमाधेनान्त्यमन्त्येन"
whose english translation is "Ādyamādyenantyamantyena."

2.4. The fourth sutra is "परावर्त्य योजयेत्"
whose english translation is "Parāvartya Yojayet".

Its meaning is "Transpose and Apply."
For example, if we divide $x^3 + 7x^2 + 6x + 5$ by $x - 2$, then

(i) x^3 Divided by x gives us x^2 which is therefore the first term of the quotient

$$\frac{x^3 + 7x^2 + 6x + 5}{x - 2}$$

$$\therefore Q = x^2 + \dots$$

(ii) $x^2 \times -2 = -2x^2$ but we have $7x^2$ in the dividend. This means that we have to get an additional $9x^2$ more. This must result from the multiplication of x by $9x$. Hence the 2nd term of the divisor must be $9x$

$$\therefore Q = x^2 + 9x + \dots$$

(iii) As for the third term, we already have $-2 \times 9x = -18x$. But, we have $6x$ in the dividend. We must therefore get an additional $24x$. Thus can only come in by the multiplication of x by 24. This is the third term of the quotient.

$$\therefore Q = x^2 + 9x + 24$$

(iv) The last term of the quotient multiplied by -2 gives us -48. But the absolute term in the dividend is 5. We have therefore to get an additional 53 from some where. But there is no further term left in the dividend. This means that the 53 will remain as the remainder

Therefore, we get $Q = x^2 + 9x + 24$ and $R = 53$.

Its Sub-sutra is "केवलैः सप्तकं गुण्यात" whose english translation is "Kevalaih Saptakam Gunȳat".

2.5. The fifth Sutra is "शून्यं साम्यसमुच्चय"
whose english translation is "Sūnyam Samyasamuccaye".
Its meaning is "When the Samuccaya is the same that Samuccaya is zero, i.e. it should be equated to zero."
For example, we consider the equation: $\frac{2x + 9}{6x + 7} = \frac{2x + 7}{2x + 9}$. Then, since the total of the numerators and the total of the denominators is the same. So, according to the Sutra that total can be equated to zero: $4x + 16 = 0$ and so $x = -4$ as the solution.

Its Sub-sutra is "वेष्टनम्" whose english translation is Vestanam.

2.6. The sixth sutra is "आनुरूप्ये शून्यमन्यत"
whose english translation is "Ānurūpye Śunyamanyat".
Its meaning is " Whatever the extent of its deficiency".
For example, we take the product 46×44. Then,

```
     46 - 04
×    44 - 06
   ────────────
  2) 40 / 24
   ────────────
     20 / 24
```

Its sub-sutra is "यावदूनं तावदूनम्" whose english translation is "Yāvadūnam Tāvadūnam".

2.7. The seventh Sutra is "सङ्कलन- व्यवकलनाभ्याम्"
whose english translation is "Sankalana Vyavakalanābhyām".
Its meaning is "By addition and by subtraction."
As an example, if we need to find the Highest Common factor (HCF) of $(x^2 + 7x + 6)$ and $x^2 - 5x - 6$. Then, $x^2 + 7x + 6 = (x + 1) (x + 6)$ and $x^2 - 5x - 6 = (x + 1) (x - 6)$.
So that the HCF is x + 1.
Its Sub-sutra is "यावदूनं तावदूनीकृत्य वर्गंचम्योजयेत"
whose english translation is "Yāvadūnam Tāvadūnīkrtya Vargañca Yojayet".

2.8. The eight sutra is "पूरणपूरणाभ्याम्"
whose english translation is "Puranāpuranābhyām"
Its meaning is "By the completion or not completion".
Its Sub-sutra is "अन्त्ययोर्दशकेडपि" whose english translation is "Antyayordasake' pi".

2.9. The ninth Sutra is "चलनकलनाभ्याम्"
whose english translation is "Sopantyadvayamantyam".
Its meaning is "The ultimate and twice the penultimate"
Its Sub-sutra is "अन्त्ययोरेव" whose english translation is "Antyayoreva".

2.10. The tenth Sutra is "यावदूनम्"

whose english translation is "Yāvadūnam"

Its meaning is " Whatever the extent of its deficiency".

For example, consider 94^2. Then, we have $(94 - 6)(6^2) = 8836$.

Its Sub-sutra is "समुच्चय गुणित" whose english translation is "Antyayoreva".

2.11. The eleventh Sutra is "व्यष्टिसमष्टिः"

whose english translation is "Vyastisamastih"

Its meaning is "Part and whole".

Its Sub-sutra is "लोपनस्थापनाभ्या" whose english translation is "Lopanasthāpanabhyām."

2.12. The twelve Sutra is "शेषाण्डैन चरमेण"

whose english translation is "Śesānyankena Caramena".

Its meaning is "The remainders by the last digit".

For example, we need to find the decimal value of 1/7. The remainders are 3, 2, 6, 4, 5 and 1. So, Multiplied by 7 these remainders give successively 21, 14, 42, 28, 35 and 7. So, ignoring the left hand side digits, we simply put down the last digit of each product and we get 1/7 = .14 28 57! Now, this 12th sutra has a subsutra *Vilokanam*. *Vilokanam* means "mere observation."

Its Sub-sutra is "विलोकनम्" whose english translation is "Vilokanam".

2.13. The thirteenth Sutra is " सोपान्त्यद्वयमन्त्य"

whose english translation is "Sopantyadvayamantyam."

Its meaning is "The ultimate and twice the penultimate".

Its Sub-sutra is "गुणितसमुच्चयः समुच्चयगुणितः"

whose english translation is "Gunitasamuccayah Samuccayagunitah."

2.14. The fourteenth Sutra is "एकन्यूनेन पूर्वेण"

whose english translation is "Ekanyūnena Pūrvena."

Its meaning is "By one less than the one before".

For example, we consider the product 763×999. Then, we get $(763 - 1)(999 - 762) = 762237$.

2.15. The fifteenth Sutra is "गुणितसमुच्चयः"

whose english translation is "Gunitasamuccayah".

Its meaning is "The product of the sum is equal to the sum of the product."

2.16. The sixteenth Sutra is "गुणकसमुच्चयः"

whose english translation is "Gunakasamuccayah."

Its meaning is "The product of the sum of the coefficients in the factors is equal to the sum of the coefficients in the product."

For example, we consider the quadratic equation $(x + 7)(x + 9) = x^2 + 16x + 63$ and we observe

that $(1 + 7)(1 + 9) = 1 + 16 + 63 = 80$. Also, we find that this rule is applicable to higher degree polynomials. Thus, if and when some factors are known this rule helps us to fill in the gaps.

3. Conclusion

Vedic mathematics is an ancient Indian mathematics discovered by ancient sages of India. It was rediscovered by Jagadguru shankaracharya Bharathi Krishna Teerthji maharaja (1884-1960) in the year 1965. Swaiji called the use of Vedic mathematics is as mental calculation. Vedic mathematics consists 16 sutras (formulae) and 13 Upa sutras (sub formulae), these sutras cannot be finding in Atharva Veda. These sutras were derived from Atharva Veda discovered by ancient sages of India. Also, Vedic mathematics is a unique system of computation based on simple rules and basic principles. By using which we can able to solve complicated mathematical calculations within few seconds. There is an interrelationship between sequence space with Vedic sutras and we find interesting examples to justify it.

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